Available online at www.sciencedirect.com
ScienceDirect
International Journal of Heat and Mass Transfer 51 (2008) 4392–4399
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Joule heating in magnetohydrodynamic flows in channels with thin conducting walls

J. Mao [1], S. Aleksandrova, S. Molokov *

Department of Mathematical Sciences, Faculty of Engineering and Computing, Coventry University, Priory Street, Coventry CV1 5FB, UK

Received 22 May 2007
Available online 7 April 2008

Abstract

Joule heating in liquid metal magnetohydrodynamic flows is investigated with reference to self-cooled liquid metal blankets for tokamaks. Pressure-driven flow of an electrically conducting fluid confined between two parallel, infinite walls with a transverse magnetic field is studied. The walls are electrically conducting, which implies strong currents flowing within the thin conducting walls. The problem is solved both analytically and numerically.

It is shown that the Joule heat cannot be neglected in certain range of parameters relevant to fusion blanket applications. The magnitude of the Joule heat released inside the channel and the walls depends on the thermal conductivity of the outside surface of the channel walls. For thermally conducting outside surface of the walls the Joule heat can become significant for high values of the Hartmann number and moderate average velocity. The effect is even more pronounced for thermally insulating outside surface of the walls. For example, for lead–lithium flow with stainless steel walls the temperature increase along the flow exceeds 200 °C over the length of the blanket, which is almost three times higher than that for thermally conducting outside surface of the walls.

The main reason for such a strong rise in temperature is the heat released inside the walls. The heat produced in the fluid region is quickly convected towards the exit from the channel. The heat released inside the walls can only leave the domain by diffusion into the fluid region and thus is accumulated along the channel length.

Keywords: Joule heating; Magnetohydrodynamics; Fusion blankets

1. Introduction

Heat transfer in magnetohydrodynamic (MHD) flows is important in many applications, such as liquid-metal blankets and divertors for fusion reactors [1–4]. One of the main aims of the blankets is to extract heat from plasma. Because of the limitations of the actual material properties and the efficiency of the electricity production, blankets must operate within relatively narrow range of temperature variation of between 100 °C and 200 °C [5]. Usually, pure lithium or lead–lithium alloys are used as coolant fluid, which flows in channels with thin walls made of stainless steel. Owing to the high electrical conductivity of liquid metals, the flow interacts with the strong, externally imposed magnetic field confining the plasma. This results in high electric currents induced in both the liquid metal and the electrically conducting walls.

The electric currents affect heat transfer in several ways. First of all, the flow pattern is altered greatly by the electromagnetic body force, and thus the convective heat transfer characteristics are changed. This effect has been studied by many authors, e.g. [1,6,7], and has been reviewed in [4,5]. Secondly, high electric currents flowing through conducting media lead to the increase of the temperature due to the Joule heating [8]. In this study we focus on the latter

* Corresponding author. Tel.: +44 24 76888601; fax: +44 24 76888585. *E-mail address:* s.molokov@coventry.ac.uk (S. Molokov).
[1] Present address: School of Mechanical Engineering, Hangzhou Dianzi University, Hangzhou 310018, PR China.

0017-9310/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2008.02.005

effect and its impact on the heat transfer in a fully developed channel flow between two parallel thin electrically conducting plates transverse to the magnetic field.

The non-isothermal MHD flow is characterised by several dimensionless parameters. The most important of them are the Hartmann number, Ha, which expresses the ratio of the electromagnetic and viscous forces, the Peclet number, Pe, which expresses the ratio of the convective and conductive heat fluxes, and the thermal and electrical wall conductance ratios. The Hartmann and Peclet numbers are assumed to be very high. In fusion blanket applications they are of the order of 10^3–10^5 and 10^3–10^4, respectively [9]. Thus the electromagnetic and convective terms dominate the problem.

Joule heating effect in MHD channel flows has been analysed in the past [1,7]. In those studies the walls have been assumed to be electrically perfectly conducting or insulating, and only moderate or low values of the Hartmann number have been considered. In both cases the conclusion was that Joule heating effect was insignificant even for very strong magnetic fields. This conclusion is adequate when the walls are electrically insulating or the magnetic field is relatively weak corresponding to low or moderate Hartmann numbers. However, the Joule heat is proportional to Ha^2, so that the effect of the finite conductivity of the walls on the fluid flow and the magnitude of the electric currents can be significant for high values of the Hartmann number [8,10]. Therefore, in fusion reactor blankets such assumptions are not justified.

The aim of this study is to evaluate the magnitude of the Joule heat in a channel with thin thermally and electrically conducting walls. The outside surface of the walls is either perfectly thermally conducting or thermally insulating. This corresponds to the physical conditions present in hybrid and self-cooled fusion blankets, respectively. Particular attention will be given to the temperature estimate at the fluid-wall interface, which is a crucial parameter for the blanket design. It will be shown that for the range of parameters relevant to fusion applications the Joule heating may be significant and therefore cannot be neglected.

2. Formulation

Consider the steady flow of a viscous, incompressible, electrically conducting fluid confined between two infinite parallel plates of thickness h_{w}^* separated by a distance of $2a^*$ (Fig. 1). The flow is subject to a uniform transverse magnetic field $\mathbf{B}^* = B_0^*\hat{\mathbf{y}}$. Here (x^*, y^*, z^*) are Cartesian coordinates. The flow is hydrodynamically fully developed (Hartmann flow). The fluid flows with mean velocity v_0^* subject to a constant pressure gradient, $\partial p^*/\partial x^* = -K^*$. Both walls are electrically and thermally conducting.

The dimensionless equations governing the flow are [1]

$$Ha^{-2}\frac{\mathrm{d}^2 u}{\mathrm{d}y^2} - j_z = -K, \tag{1}$$

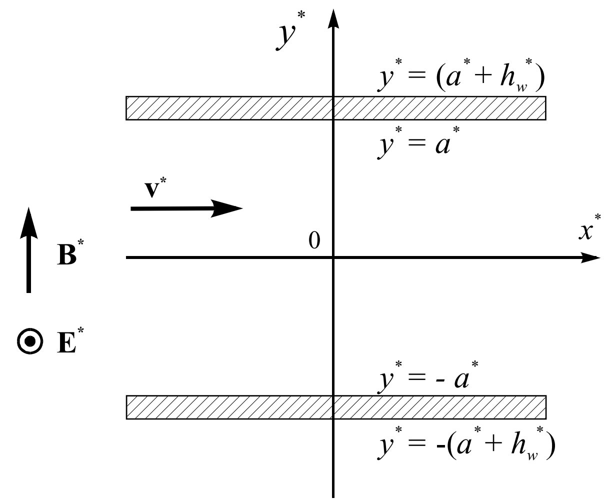

Fig. 1. Schematic diagram of the Hartmann flow.

$$j_z = E + u, \tag{2}$$

$$\nabla^2 T + j_z^2 + Ha^{-2}\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2 = Pe \cdot u\frac{\partial T}{\partial x}. \tag{3}$$

Here the length, the fluid velocity $\mathbf{v} = u(y)\hat{\mathbf{x}}$, the pressure p, the electric current density $\mathbf{j} = j_z\hat{\mathbf{z}}$, the electric field $\mathbf{E} = E\hat{\mathbf{z}}$, and the temperature T are normalized by a^*, v_0^*, $a^*\sigma^* v_0^* B_0^{*2}$, $\sigma^* v_0^* B_0^*$, $v_0^* B_0^*$ and $(T^* - T_0^*)/\Delta T^*$, respectively; $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$. The characteristic temperature difference $\Delta T^* = a^{*2}\sigma^* v_0^{*2} B_0^{*2}/\lambda^*$ is based on the Joule dissipation. The square of the Hartmann number, $Ha = a^* B_0^*\sqrt{\sigma^*/\rho^* \nu^*}$, characterises the ratio of the electromagnetic to the viscous forces. The Peclet number, $Pe = v_0^* a^* \rho^* C_p^*/\lambda^*$, determines the ratio of convective and conductive heat fluxes. In the above, ρ^*, ν^*, λ^*, σ^* and C_p^* are the fluid density, kinematic viscosity, thermal conductivity, electrical conductivity, and specific heat of the fluid, respectively.

The dimensionless equations governing the electric current density and the temperature in the walls are as follows:

$$j_{z\mathrm{w}} = \sigma_{\mathrm{w}} E, \tag{4}$$

$$\nabla^2 T_{\mathrm{w}} + \frac{j_{z\mathrm{w}}^2}{\sigma_{\mathrm{w}}\lambda_{\mathrm{w}}} = 0. \tag{5}$$

Here T_{w} is the wall temperature, $\lambda_{\mathrm{w}} = \lambda_{\mathrm{w}}^*/\lambda^*$ and $\sigma_{\mathrm{w}} = \sigma_{\mathrm{w}}^*/\sigma^*$ are relative thermal and electrical conductivities of the walls.

We take an advantage of the symmetry of the problem and construct a solution for $y > 0$ (top half of the channel) subject to the symmetry conditions

$$\frac{\partial u}{\partial y} = 0, \quad \frac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = 0. \tag{6a, b}$$

The total electric current passing through a cross-section $x = $ constant must vanish. Integrating the electric current in the fluid and in the walls defined by Eqs. (2) and (4) yields

$$\int_0^1 (E+u)\mathrm{d}y + \int_0^h \sigma_\mathrm{w} E \mathrm{d}y = 0, \tag{7}$$

where $h = h_\mathrm{w}^*/a^*$ is the dimensionless wall thickness. The dimensionless mass flux is equal to 2, i.e.

$$\int_0^1 u\mathrm{d}y = 1. \tag{8}$$

The boundary condition for the fluid velocity at the wall is the no-slip condition

$$u = 0 \quad \text{at} \quad y = 1. \tag{9}$$

The outside walls are either thermally conducting, so that

$$T_\mathrm{w} = 0 \quad \text{at} \quad y = 1 + h, \tag{10a}$$

or thermally insulated, so that

$$\frac{\partial T_\mathrm{w}}{\partial y} = 0 \quad \text{at} \quad y = 1 + h. \tag{10b}$$

Finally, the temperature and the normal heat flux are continuous at the interface between the wall and the fluid, which yields

$$T_\mathrm{w} = T, \quad \lambda_\mathrm{w}\frac{\mathrm{d}T_\mathrm{w}}{\mathrm{d}y} = \frac{\mathrm{d}T}{\mathrm{d}y} \quad \text{at} \quad y = 1. \tag{11a, b}$$

The fluid velocity is obtained independently of the temperature from Eqs. (1), (2), (4), (7), (8) and (9) to give the well-known Hartmann profile [4]:

$$u = \frac{Ha}{Ha - \tanh Ha}\left[1 - \frac{\cosh(Hay)}{\cosh(Ha)}\right], \tag{12}$$

while the pressure gradient is

$$K = \frac{cHa + \tanh(Ha)}{(1+c)(Ha - \tanh(Ha))}, \tag{13}$$

where $c = \sigma_w^* h_w^*/\sigma^* a^*$ is the wall conductance ratio.

The electric field is determined from Eqs. (7), (8) and is equal to

$$E = -\frac{1}{1+c}. \tag{14}$$

Substituting Eq. (14) into Eq. (4) yields the electric current in the wall as follows:

$$j_{z\mathrm{w}} = -\frac{\sigma_\mathrm{w}}{1+c}. \tag{15}$$

The electric current in the fluid is defined by Eq. (2).

3. Thermally conducting outer wall surface

If the temperature of the outside surface of the walls is fixed ($T = 0$ at $y = 1 + h$), as e.g., in hybrid blankets, a solution independent of x is possible. In this case the flow is assumed to be thermally fully developed, and Eqs. (3) and (5) subject to the conditions (6), (10a) and (11) can be solved exactly. An analytical solution for the temperature in the flow and in the thin walls is

$$T = D_1\left[\frac{1}{2}\left(1 - \frac{\cosh^2(Hay)}{\cosh^2(Ha)}\right) - \frac{2D_2}{Ha}\left(1 - \frac{\cosh(Hay)}{\cosh(Ha)}\right) + \frac{1}{2}D_2^2\left(1 + \frac{2h}{\lambda_\mathrm{w}} - y^2\right) - D_3\frac{\tanh(Ha)h}{(1+c)\lambda_\mathrm{w}}\right] + \frac{hc}{2\lambda_\mathrm{w}(1+c)^2}, \tag{16}$$

$$T_\mathrm{w} = \frac{1}{2}D_4[(1+h)^2 - y^2] - (1+h-y) \times \left[D_4 + \frac{D_1}{\lambda_\mathrm{w}}\left(D_3\frac{\tanh(Ha)}{1+c} - D_2^2\right)\right], \tag{17}$$

where

$$D_1 = \frac{1}{[Ha - \tanh(Ha)]^2}, \quad D_2 = \frac{cHa + \tanh(Ha)}{1+c}, \quad D_3 = -Ha + cHa + 2\tanh(Ha), \quad D_4 = \frac{\sigma_\mathrm{w}}{(1+c)^2\lambda_\mathrm{w}}. \tag{18}$$

If the channel walls are thermally perfectly conducting ($\lambda_\mathrm{w} \to \infty$), limits $c \to 0$ or $c \to \infty$ in Eq. (16) give the solutions for a channel with electrically insulated or perfectly conducting walls, respectively, presented in [1].

A typical non-dimensional temperature profile for finite values of λ_w and c is shown in Fig. 2. As the electric current is higher in the thin conducting walls than that in the flow, the temperature gradient inside the walls is much higher than that in the flow.

From Eqs. (5) and (15) follows that Joule dissipation in the walls has a maximum precisely at $c = \sigma_\mathrm{w} h = 1$. Fig. 3 shows temperature dependence on the electrical conductivity and thickness of the wall for $c < 1$, typical for blankets. In this range, higher electrical conductivity of the walls results in higher wall currents so that more Joule heat is released into the channel (Fig. 3a). With σ_w changing from 1 to 2, the corresponding temperature at the wall doubles. Similar effect takes place if the wall thickness is increased (Fig. 3b). Note that only values of h below 0.1 are of

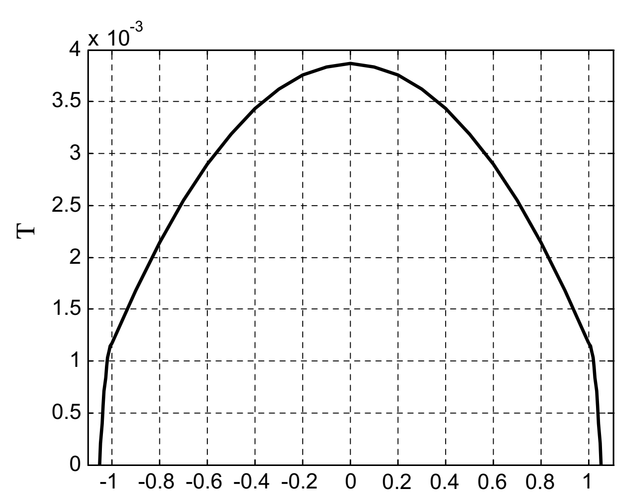

Fig. 2. Dimensionless temperature for thermally conducting outside wall surface and for $h = 0.05$, $\sigma_\mathrm{w} = 1.58$, $\lambda_\mathrm{w} = 1.68$, $Ha = 5820$.

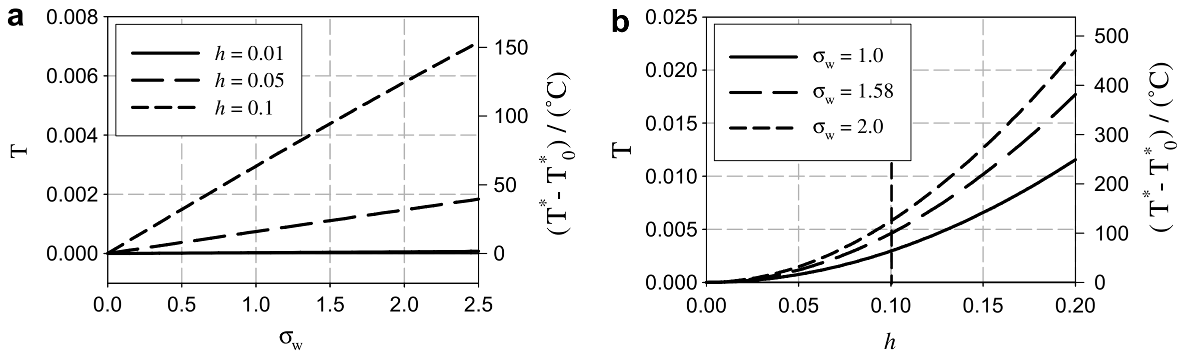

Fig. 3. Dimensionless/dimensional temperature variation with (a) electrical conductivity of the wall and (b) thickness of the wall for thermally conducting outside wall surface at symmetry plane $y^* = 0$ for $a^* = 0.06$ m, $v_0^* = 1$ m/s, $B_0 = 10$ T ($Ha = 11{,}640$).

practical interest in fusion applications (to the left of the vertical broken line).

In a strong magnetic field the non-dimensional temperature defined by Eq. (16) becomes

$$T \to \frac{c^2(1-y^2)}{2(1+c)^2} + \frac{c(2c+1)}{2Bi(1+c)^2} \quad \text{as} \quad Ha \to \infty. \tag{19}$$

Here $Bi = \lambda_w/h$ is the Biot number, which describes the relative thermal conductivity of the wall in the normal direction. As there is no convective heat transfer, Joule heat is transported by diffusion, and is removed at the outside wall surface. This results in lower temperatures near the walls with temperature maximum in the centre of the channel.

The dimensional temperature grows proportionally to $B_0^{*2}v_0^{*2}$ and thus can reach high values in high magnetic fields or for high flow velocities. For a flow of lead–lithium [4] with stainless steel walls [11] ($\lambda^* = 13.184$ W/mK, $\rho^* v^* = 2.097 \times 10^{-3}$ kg/ms, $\lambda_w^* = 22.2$ W/mK, $\sigma^* = 7.891 \times 10^5 (\Omega\,\text{m})^{-1}$, $\sigma_w^* = 1.25 \times 10^6 (\Omega\,\text{m})^{-1}$) in a channel of width $a^* = 0.06$ m, walls of thickness $h_w^* = 0.003$ m, magnetic field of 5 T, and average velocity of $v_0^* = 2$ m/s, the maximum temperature in the mid-plane of the flow is 83 °C. This is a sufficiently high value which may be reached in dual-coolant blankets with thin conducting walls. In hybrid blankets, such as European Helium Cooled Lead Lithium (HCLL) concept fluid velocities are much lower, of the order of mm/s [12], so that Joule heating effect becomes insignificant.

In self-cooled blankets, however, channel walls normally have thermally insulating outer wall surface. This case will be considered in next section.

4. Thermally insulating outer wall surface

When the outside surface of the walls is thermally insulating, the heat released in the flow cannot leave the channel through the walls, and thus no thermally fully developed flow exists. In order to study the Joule heating effect in such a configuration, we consider developing temperature in a long channel. The temperature is fixed at the entrance to the channel:

$$T = 0 \quad \text{at} \quad x = 0. \tag{20}$$

It has been shown [7] that for channels with electrically insulating walls viscous dissipation and Joule dissipation are of the same order. When the electrical conductivity of the channel walls increases, the electric currents within the walls, and thus the Joule dissipation become stronger. However, viscous dissipation does not depend on c and thus remains constant. Therefore, the Joule effect becomes dominant in channels with walls of finite conductivity. This allows one to neglect viscous effects in Eq. (3) and to use slug flow approximation ($u = 1$) for flows with moderate to high Hartmann number.

4.1. Numerical solution

Eqs. (3) and (5) subject to the boundary conditions (2), (6), (10b), (11), (14) and (15) have been solved numerically using CFX. This is a commercial fluid dynamics package based on the finite volume technique and the SIMPLE (semi-implicit method for pressure-linked equations) family algorithms for the pressure–velocity coupling.

All calculations have been performed for liquid lead–lithium flow with the stainless steel walls. Other dimensional parameters are: $a^* = 0.06$ m, the thickness of the wall $h^* = 0.003$ m, the axial distance $x_{\max}^* = 10.8$ m, $v_0^* = 2$ m/s, the magnetic field varying between 1 T and 5 T. The corresponding dimensionless parameters are: $c = 0.079$, $Pe = 16396$, $Ha = 1164$, 2328, 3492, 4656, 5820. The dimensionless length of the channel is $x_{\max} = 180$, which approximately corresponds to the total length of the blanket.

The mesh used for calculations is uniform in the axial direction and non-uniform in the transverse direction with points clustered near the wall. The smallest mesh size in the transverse direction is 0.01. The results have been found to be sensitive to the axial mesh resolution. The axial mesh size of 0.25 was sufficient to achieve excellent agreement with the analytical solutions for the whole range of parameters considered here. An example of the comparison is presented in Fig. 4. In all cases, calculations stopped when the residual in the energy equation was smaller than 10^{-8}.

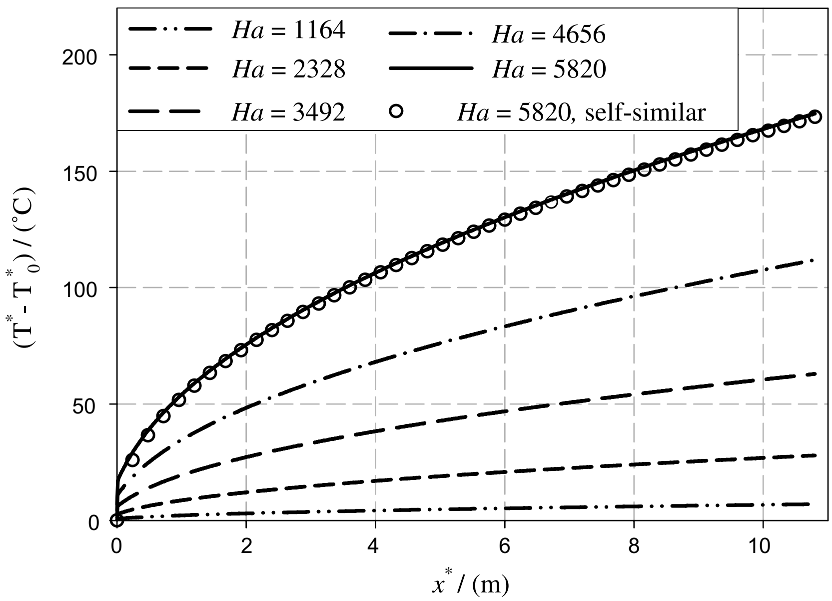

Fig. 4. Variation of dimensional temperature with the axial length for thermally insulating outside wall surface at the fluid-wall interface for $a^* = 0.06$ m, $h^* = 0.06$ m ($h = 0.05$), $v_0^* = 2$ m/s ($Pe = 16{,}396$): numerical solution (lines) and self-similar solution (circles).

The development of temperature with the axial distance at the fluid-wall interface and at the channel axis is shown in Figs. 4 and 5, respectively. Fig. 4 shows that the temperature at the fluid-wall interface increases with the Hartmann number significantly. At the exit from the channel it reaches a value of 175 °C for $Ha = 5820$, corresponding to the magnetic field of 5 T. Fig. 5 shows that the temperature at the channel axis does not rise significantly even for high values of the Hartmann number.

The main reason why the Joule heating is strong at the fluid-wall interface is the following. As the Peclet number is high, the whole blanket operates in a thermally developing regime. The heat released inside the fluid is convected to the exit from the domain very quickly. However, there is no convective heat transfer inside the walls. The only possibility for the heat to leave the domain is to diffuse into the fluid, where it will be transported by convection towards the exit. As the diffusion is a slow process, high

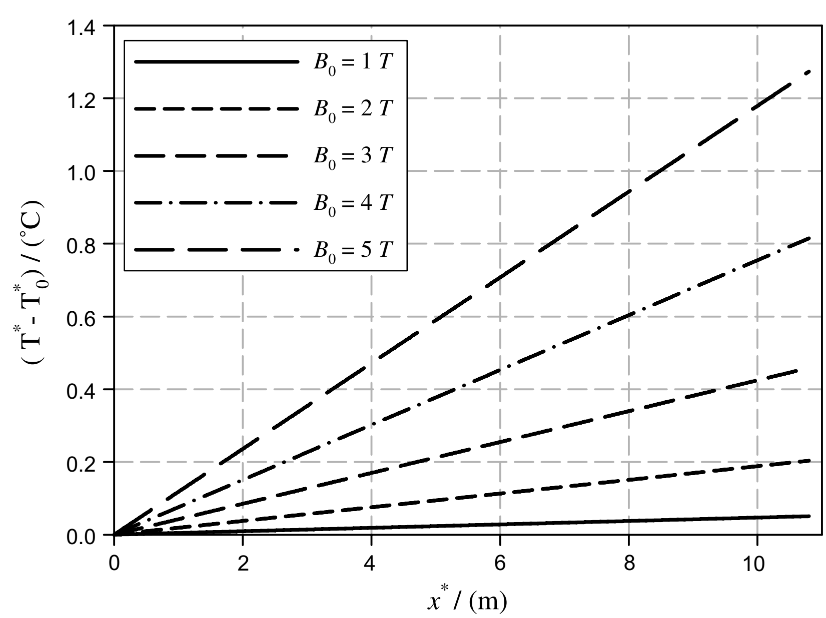

Fig. 5. Variation of dimensional temperature with the axial length for thermally insulating outside wall surface at the axis $y^* = 0$ for $a^* = 0.06$ m, $h^* = 0.06$ m ($h = 0.05$), $v_0^* = 2$ m/s ($Pe = 16{,}396$): numerical solution.

peaks of temperature appear both inside the walls and in the fluid near the walls, where the thermal boundary layers are formed.

4.2. Self-similar solution

When the walls of the channel are sufficiently thin, the temperature gradient inside the walls is much higher in the y-direction than that in the x-direction. Therefore, the term $\partial^2 T_w/\partial x^2$ in Eq. (5) can be neglected yielding equation

$$\frac{\partial^2 T_w}{\partial y^2} = -\frac{\sigma_w}{\lambda_w (1+c)^2}. \tag{21}$$

Integrating Eq. (21) twice and using boundary conditions (10b) yields

$$T_w = \frac{\sigma_w}{\lambda_w (1+c)^2}\left[(1+h)y - \frac{y^2}{2}\right] + C(x), \tag{22}$$

where $C(x)$ is an unknown function.

Next, for high values of the Peclet number, the term $\partial^2 T/\partial x^2$ in Eq. (3) is negligible so that Eq. (3) is reduced to

$$\frac{\partial^2 T}{\partial y^2} + \frac{c^2}{(1+c)^2} = Pe\frac{\partial T}{\partial x}. \tag{23}$$

The matching conditions (11b) and Eq. (22) then provide the boundary conditions for the temperature in the fluid:

$$\frac{dT}{dy} = \frac{c}{(1+c)^2} \quad \text{at} \quad y = 1. \tag{24}$$

For $\xi = x/Pe \ll 1$, the solution to Eq. (23) subject to boundary condition (24) and symmetry condition (6) may be sought in the following form:

$$T = \frac{c^2}{(1+c)^2}\xi + t(\xi, y) + t(\xi, -y), \tag{25}$$

Function $t(\xi, y)$ is expressed in a self-similar form as follows:

$$t = \sqrt{\xi}F(\eta), \tag{26}$$

where $\eta = (1-y)/\sqrt{\xi}$ and

$$F(\eta) = \frac{c}{(1+c)^2}\left\{\frac{2}{\sqrt{\pi}}\exp(-\eta^2/4) - \eta erfc(\eta/2)\right\}. \tag{27}$$

The first term in Eq. (25) represents temperature in the core. The other two terms represent thermal, developing boundary layers of thickness $O(Pe^{-1/2})$ at walls $y = 1$ and $y = -1$, respectively. They are equivalent to boundary layers at semi-infinite walls with a constant heat flux [13]. For $\xi \ll 1$, which holds in fusion blanket applications, the effect of each boundary layer on the one at the opposite wall, as well as on the core, is exponentially small.

Now, substituting Eqs. (25)–(27) into the boundary condition (11a) and neglecting exponentially small terms as $Pe \to \infty$ gives

$$C(x) = \frac{c^2}{(1+c)^2}\frac{x}{Pe} + \frac{c}{(1+c)^2}\left\{2\sqrt{\frac{x}{\pi \cdot Pe}} - \frac{1}{2\lambda_w h}(1+2h)\right\}, \tag{28}$$

so that Eq. (22) yields

$$T_w = \frac{c^2}{(1+c)^2}\xi + \frac{c}{(1+c)^2}\left\{\frac{2\sqrt{\xi}}{\sqrt{\pi}} - \frac{1}{2\lambda_w h}(y-1)[y-(1+2h)]\right\}. \tag{29}$$

Comparison with the numerical solution for lead-lithium flow with stainless steel walls for $B_0^* = 5$ T ($Ha = 5820$) is given in Fig. 4. It is evident that the self-similar solution gives a perfectly adequate estimate of temperature variation for flows at high values of the Peclet number and for thin conducting walls.

At each cross-section $x = $ constant the temperature is highest at the outer wall surface,

$$T_w = \frac{c^2}{(1+c)^2}\xi + \frac{2c\sqrt{\xi}}{(1+c)^2\sqrt{\pi}} + \frac{c}{(1+c)^2}\frac{h}{2\lambda_w} \quad \text{at} \quad y = 1+h. \tag{30}$$

At the fluid-wall interface,

$$T_w = \frac{c^2}{(1+c)^2}\xi + \frac{2c\sqrt{\xi}}{(1+c)^2\sqrt{\pi}} \quad \text{at} \quad y = 1. \tag{31}$$

The first terms, proportional to ξ, in Eqs. (29)–(31) dominate in the region $\xi \gg 4/\pi c^2$, where the temperature exhibits the linear profile corresponding to the fully developed regime. For high values of the Peclet number characteristic to the fusion blankets this condition is not satisfied, and the fully developed flow regime is never reached.

For small values of ξ ($\xi \ll 1$) the first term becomes negligible provided $c < 2/\sqrt{\pi}$. The third term in Eq. (30) is important only at the entrance to the channel ($\xi < \pi h^2/(16\lambda_w^2)$). Therefore, the temperature inside the wall develops as $\sqrt{\xi}$ along the whole length of $x_{max} = 180$. In dimensional variables, at the exit from the channel this dominant term gives the estimate of maximum temperature as follows:

$$T_{max}^* = A\frac{a^*\sigma^* v_0^{*3/2} B_0^{*2}}{\sqrt{\lambda^*\rho C_p}}\sqrt{x_{max}^*} \tag{32}$$

and thus grows as $\sim B_0^{*2} v_0^{*3/2}$. Here $A = 2c/(1+c)^2\sqrt{\pi}$.

Temperature profiles at different distances from the entrance to the channel (Fig. 6) show diffusion of heat generated inside the walls into the flow region as ξ increases.

If the walls are *perfectly conducting*, the temperature in the flow defined by Eq. (25) becomes linear with respect to ξ and independent of y, namely,

$$T \to \xi \quad \text{as} \quad c \to \infty. \tag{33}$$

As there is no Joule heating inside the walls, there is no heat flux into the flow region from the walls. In the flow region, the Joule heat is proportional to u^2 and is almost uniform. It is convected in the x-direction with no diffusion in

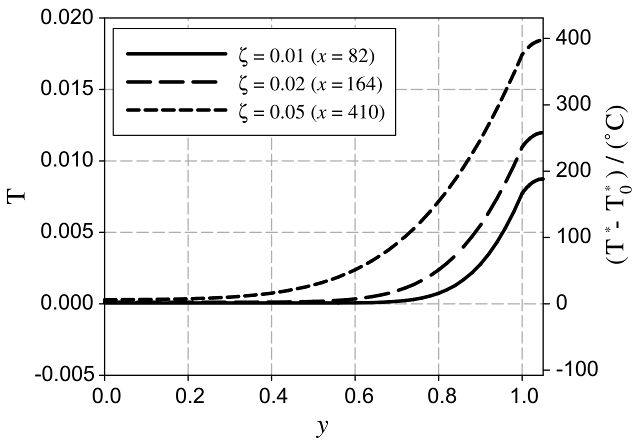

Fig. 6. Variation of dimensionless/dimensional temperature with y for thermally insulating outside wall surface for $a^* = 0.06$ m, $h^* = 0.06$ m ($h = 0.05$), $v_0^* = 2$ m/s ($Pe = 16,396$), $B_0 = 5$ T ($Ha = 5820$): self-similar solution.

the y-direction taking place. The dimensional temperature in a channel with electrically perfectly conducting walls is

$$T^* = T_0^* + \frac{\sigma v_0^* B_0^{*2}}{\rho^* C_p^*} x^*.$$

For lead–lithium flowing with $v_0^* = 2$ m/s in a channel with half-width $a^* = 0.06$ m in a magnetic field of $B_0^* = 5$ T, the increase in temperature over 200 values of the characteristic length is 261 °C.

For *finite values of the wall conductance ratio c*, the temperature can exceed that for electrically perfectly conducting walls defined by Eq. (33). Fig. 7 shows temperature at the fluid-wall interface for several values of c. Compared to channel with $c = 0.079$, the fully developed temperature (33) characteristic for perfectly electrically conducting walls is higher if the channel length is $\xi_{\max} > 0.006$. However, for higher c the temperature at the interface will exceed $T = \xi$ in the whole length of channel.

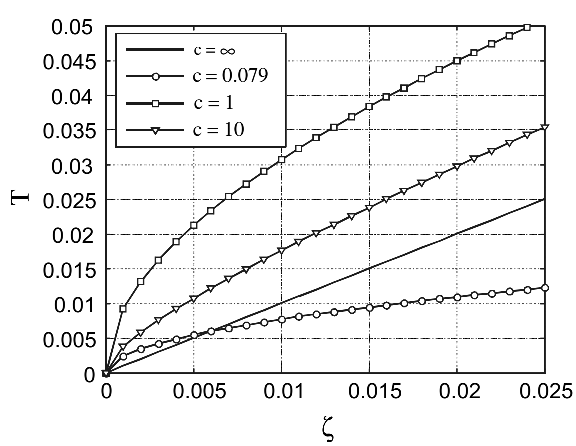

Fig. 7. Variation of dimensionless temperature with ξ at the fluid-wall interface for $a^* = 0.06$ m, $h^* = 0.06$ m ($h = 0.05$), $v_0^* = 2$ m/s ($Pe = 16,396$), $B_0 = 5$ T ($Ha = 5820$): self-similar solution.

It follows from Eq. (25) that for small values of c and ξ the dimensional heat flux into the channel from the walls caused by the Joule heat is

$$q_w^* = \lambda^* \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=a^*} = \frac{\lambda^* \Delta T^*}{a^*} \frac{c}{(1+c)^2} = h_w^* \sigma_w^* v_0^{*2} B_0^{*2}.$$

For lead–lithium flowing with $v_0^* = 2$ m/s in a channel with stainless steel walls of thickness $h^* = 0.003$ m in a magnetic field of $B_0^* = 5$ T, the heat flux is $q_w = 0.375$ MW/m^2. It is much higher than that presented for electrically insulating or perfectly conducting walls [7].

5. Conclusions

Hartmann flow with thin electrically conducting walls has been studied both analytically and numerically. It has been shown that for the thermally conducting outside surface of the walls the Joule heating can become significant for high values of the Hartmann number and moderate average velocity.

When the outside surface of the walls is thermally insulating, the effect is even more pronounced. Under the same conditions as above, the temperature can rise by over 200 °C over a distance of about 180 values of the characteristic length. This increase is caused by strong electric current flowing inside the domain. While in the flow region the released heat is convected out of the domain, there is no convection inside the walls. Therefore, the heat is accumulated in thin conducting walls and is partly diffused into the fluid.

In previous studies [1,7] the Joule heating effect has been found insignificant, even for high values of the Hartmann number and high velocities. These studies differ with the present investigation in that the Joule heat released in the walls of finite electrical conductivity and thickness has been considered here. It has been shown in this study that this is a major effect, which needs to be taken into account for the design of self-cooled blankets for fusion reactors.

Present study dealt with a simple theoretical model of the Joule heating in channel flow. However, the presence of the sidewalls in the channel with a rectangular cross-section will modify the fluid flow, and therefore the heat transfer. Depending on the parameters, the electric current may complete its loop inside the sidewalls [10]. This can lead to high-velocity jets near the sidewalls, which will affect the heat transfer. However, the electric currents flowing inside the sidewalls will produce additional Joule heat. Therefore, the channel flow problem considered in this paper is the first step in understanding the nature and the magnitude of the Joule heating effect in fusion applications.

Finally, as variation of the temperature in many cases spans 200 °C, material properties such as electrical conductivity, kinematic viscosity and density may significantly vary with temperature depending on the liquid metal and wall material. As we were concerned with PbLi in this paper, such a variation is insignificant. For example, the electrical conductivity of PbLi varies by less than 6% over

200 °C. For other liquid metals, such as Na, this variation may reach 50%. In that case variation of parameters with temperature cannot be neglected and will lead to nonlinear coupling of fluid flow and heat transfer, which is currently being investigated.

References

[1] E. Blums, Yu. A. Mikhailov, R. Ozols, Heat and Mass Transfer in MHD Flows, World Scientific Publishing Co. Pvt. Ltd., 1986.

[2] L. Barleon, U. Burr, R. Stieglitz, M. Frank, K.J. Mack, MHD flow and heat transfer in a rectangular duct, Forschungszentrum Karlsruhe, Report No. FZKA5927, 1998.

[3] R. Moreau, Magnetohydrodynamics, Kluwer Academic Publishers, London, 1990.

[4] U. Müller, L. Bühler, Magnetofluiddynamics in Channels and Containers, Springer, 2001.

[5] S. Malang, M.S. Tillack, Development of self-cooled liquid metal breeder blankets, Forschungszentrum Karlsruhe, Report No. FZKA5581, 1995.

[6] L. Bühler, Convective-diffusive transport in laminar MHD flows, Forschungszentrum Karlsruhe, Report No. KfK 5241, 1993.

[7] J. Lahjomri, K. Zniber, A. Oubarra, A. Alemany, Heat transfer by laminar Hartmann's flow in thermal entrance region with uniform wall heat flux: the Graetz problem extended, Energy convers. Manage. 44 (2003) 11–34.

[8] S. Yu. Smolentsev, A.V. Tananaev, D.A. Dajeh, Laminar heat transfer in the MHD flow in a rectangular duct 1. Numerical and analytical investigations, Magnetohydrodynamics 33 (2) (1997) 148–154.

[9] S. Molokov, C.B. Reed, Liquid metal MHD flows in circular ducts at intermediate Hartmann numbers and interaction parameters, Magnetohydrodynamics 39 (2003) 539–546.

[10] J. Mao, Joule Heat in magnetohydrodynamic duct flows, Ph.D. Thesis, Coventry University, Coventry, UK, 2006.

[11] E.A. Brandes, G.B. Brook, Smithells Metals Reference Book, Butterworth Heinemann, 1992 (Chapter 14).

[12] A. Li Puma, J.L. Berton, B. Branas, L. Bühler, J. Doncel, U. Fischer, W. Farabolini, L. Giancarli, D. Maisonnier, P. Pereslavtsev, S. Raboin, J.-F. Salavy, P. Sardain, J. Szczepanski, D. Ward, Breeding blanket design and systems integration for a helium-cooled lithium–lead fusion power plant, Fusion Eng. Des. 81 (2006) 469–476.

[13] Al. M. Morega, A. Bejan, Heatline visualisation of forced convection laminar boundary layers, Int. J. Heat Mass Transfer 36 (1993) 3957–3966.